Scientific Measurement and Significant Figures

KEY POINTS:

1. Observations are quantified by using scientific instruments or equipment to make measurements, e.g., mass and volume.
2. Every measurement has uncertainty associated with it.
3. The uncertainty of a measurement is reflected by the number of significant figures. The last significant digit is the uncertain digit.
4. Calculated results must reflect the uncertainty in the measurement (data collected).

Quantitative observations involve scientific measurement. For example, a ruler and a scale are used to make quantitative measurements of substances. However, each measurement has uncertainty associated with it. The amount of uncertainty in a measurement is reflected by the number of significant figures that is reported in the measurement. When you measure an object, you want to determine the digits in the measurement that you are certain about plus one additional digit which you are allowed to guess at. This last rightmost digit in a number is the digit that is uncertain. The number of significant figures in a number tells us something about the accuracy of the measurement.

For example, you eating a foot long sandwich and want to know if it is really a foot long. You use a ruler to measure the length of the sandwich as shown in Figure 1.

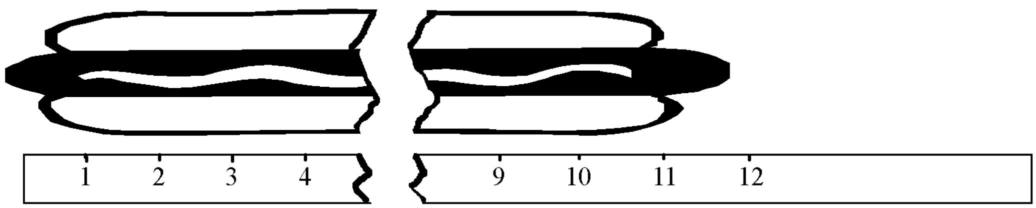

Figure 1. Measuring the length of a sandwich with a ruler and determining the number of significant figures.

From Figure 1, note that the sandwich is between 11 and 12 inches long. You know the number "11" with certainty since this number is explicitly marked on the ruler. In scientific measurement, you are allowed to guess at one additional digit. So you report the length of the sandwich as 11.5 inches long. This number has a total of three significant figures. The digit "5" is the uncertain digit that you are allowed to guess and is the last significant digit. By reporting the tenths digit, you are implying that the ruler is accurate to ±0.1 inches or ±0.2 inches depending on your ability to "eyeball" between the 11 and 12 marks on the ruler.

If you report the length of the sandwich as 11 inches, you did not guess your one allowed uncertain digit and have not reported enough significant figures. By reporting only two significant figures, you are saying the accuracy of the ruler is ±1 inch. This ruler is more accurate than ±1 inch. If you report the length of the sandwich as 11.56 inches, you have guessed at two digits and have reported too many significant figures. By reporting four significant figures, you are saying the accuracy of the ruler is ±0.01 inches and this ruler is not this accurate.

Science and chemistry use computers and hand-held calculators extensively. These instruments display many digits in numbers so it is easy to include too many significant figures in your answer. The following rules will help you determine the number of significant figures and how to round numbers:

1. zeros that are in between non-zero digits are considered significant. E.g., 2.003 has 4 significant figures.

2. For numbers that have a decimal point,

a. all zeros to the right of the last non-zero digit are significant. E.g., 2.0030 has 5 significant figures.

b. All zeros to the left of the first non-zero digit are not significant. E.g., 0.020030 has 5 significant figures.

3. For numbers that do not have a decimal point, all zeros to the right of the last non-zero digit are not significant. E.g., 20030 has 4 significant figures.

4. When converting numbers between the expanded (regular) notation and scientific notation, keep the same number of significant figures in each notation. E.g., 20030 = 2.003×10^4. Each number has 4 significant figures.

5. For mathematical operations, a calculated result is no better than the experimental data from which it came. Calculated results will have to be rounded to reflect the significant figures in the quantitative measurements.

a. Rounding numbers:

- if the discarded digit is greater than 5, increase the last retained digit by one. E.g., 15.7 (3 significant figures) rounds to 16 (2 significant figures).
- If the discarded digit is less than 5, leave the last retained digit unchanged. E.g., 15.4 (3 significant figures) rounds to 15 (2 significant figures).
- If the discarded digit is equal to 5, increase the last retained digit by one if this digit is an odd number or leave it unchanged if it is an even number. E.g., 15.5 (3 significant figures) rounds to 16 (2 significant figures) or 14.5 (3 significant figures) rounds to 14 (2 significant figures).

b. Addition and subtraction. The number of decimal places in the numbers that are being added or subtracted determines the number of significant figures in the answer. The answer will have the same number of decimal places as the number with the fewest decimal places that is being added or subtracted. The number with the fewest decimal places reflects the least accurate measurement. E.g.,

	22.2 cm	one decimal place - least accurate measurement
+	11.67 cm	two decimal places
	33.87 cm	answer needs to be rounded to one decimal place = 33.9 cm.

c. Multiplication and division. The product or quotient will have the same number of significant figures as the factor with the fewest number of significant figures. E.g.,

	14.0	three significant figures
x	6.000	four significant figures
	84.0	answer has three significant figures

d. Combining operations in a series of calculations. To avoid rounding errors, carry through all the digits in intermediate calculation steps and then round your final answer. Use the addition/subtraction and multiplication/division rules to determine the number of significant figures. E.g.,

22.2 / 14 + 6.000 = ?

22.2/14 = 1.5|857143 the digits in the intermediate answer to the left of the line between the 5 and 8 are the significant figures

1.5|857143 + 6.000 = 7.5|857143 rounded to 7.6

To summarize, every measurement has uncertainty associated with it. The uncertainty is reflected in the last significant digit (the uncertain digit or the digit with which you are allowed to guess). By looking at the measuring device you are using, you can determine the digits you know with certainty with the next digit being the uncertain digit. The sum of the digits you know with certainty plus the uncertain digit gives you the number of significant figures. The uncertain digit tells you the sensitivity of the measuring device.

Random Error and Systematic Error

Each measurement has uncertainty associated with it, i.e., each measurement has error. Error refers to the numerical difference between a measured value and the true value. There are two types of errors: *random* error and *systematic* error.

You take 10 g of sand and weigh it 10 times. If you use a coarse mass measuring device, such as a triple beam balance that measures mass to the nearest 1 g, each measurement should give you the same result each time. In other words, your 10 measurements are reproducible. However, if you used a more sensitive balance, such as an analytical balance, each mass measurement will of slightly different in the last digit. These random fluctuations in the measured quantity are called *random* error. Random error is caused by unpredictable and imperceptible factors that are beyond the control of the experimenter, i.e., you.

Errors that are due to definite causes are called *systematic* errors. A systematic error is, in general, reproducible and always higher than the true value or always lower than the true value. In many cases, a systematic error can be predicted or identified by a person who thoroughly understands all the aspects of the measurement. Examples of sources of systematic errors include a corroded weight, parallax reading of a buret, a poorly calibrated buret, an impurity in a reagent, an appreciably solubility of a precipitate, a side reaction in a titration, and heating a sample at too high a temperature.

During the course of Chem 1B lab, you will often compare your experimental result to a true value. Random errors are always present but you want to reduce or eliminate systematic errors in your experimental measurements.

Accuracy and Precision

Since each measurement has uncertainty associated with it, we will determine how "good" our measurements and experiments are. Error in measurement is reflected in accuracy and precision.

Recall the last time you played darts. A throw that is very close to the bull's eye is accurate. A set of throws that is spread all over the board is not precise. Accuracy refers to the closeness of an experimental value to its "true" value. Precision refers to the closeness of a set of data to each other. Quantitatively, accuracy is represented by absolute error and percent error. Absolute error is the difference between the experimental value and the true value:

$$\text{Absolute error} = \text{experimental value} - \text{true value} \qquad (1).$$

The percent error is the absolute error relative to the true value:

$$\%\ \text{error} = \frac{\text{absolute error}}{\text{"true" value}} \times 100 \qquad (2).$$

In science, we want our observations to be reproducible, i.e., we want to get the same result each time to tell us that what we are seeing is what we want to see. Precision can be quantified by calculating the % difference:

$$\%\ \text{difference} = \frac{\text{high - low}}{\text{average}} \times 100 \qquad (3).$$

There are other ways to measure precision of a set of results: average deviation and % average deviation, standard deviation and % standard deviation.

Table 1 lists the uncertainties of various measuring devices. The uncertainties are expressed in the significant figures that the device is capable of measuring.

Table 1. Uncertainties of Various Measuring Devices

Measuring Device	Uncertainty
12 cm ruler	± 0.05 cm
Triple beam balance	± 0.05 g
Analytical balance	± 0.0001 g
10 ml graduated cylinder	± 0.05 ml
100 ml graduated cylinder	± 0.5 ml
50 ml buret	± 0.02 ml
25 ml volumetric flask	± 0.02 ml
25 ml transfer pipet	± 0.02 ml

With your knowledge of scientific measurement, the next time someone asks you how much you weigh, respond qualitatively ("a little" or "a lot") or quantitatively ("50" and remember those units unless you have ulterior motives).

Reference:

1. R.A. Day and A. L. Underwood, "Quantitative Analysis", 5th ed., Chapters 1 and 2, Prentice Hall, 1986.